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# White Paper

# Valid RF Field Measurements using Lee Sampling Criteria

William Lee published his pioneering paper on RF signal level sampling in February 1985 [1]. This article explains the background and thinking of his analysis without the fundamental mathematics and also references this with more recent work by David Parsons [2] suggesting higher sample numbers.

Lee's figure for valid average power measurements of 36 samples over 40  $\lambda$  (called "Lee-criteria") should not be treated as an absolute, rather as a guide line. It should be reviewed in the light of the measurements to be made and that other factors should be considered that can affect the numbers of samples needed, such as whether the receiver has a linear or logarithmic characteristic.

Willtek have been supporting the wireless industry with the Griffin Series of Fast Measurement Receivers. Griffins have the ability to accurately sample RF signal levels at high speed (up to 100,000 samples/sec) to allow drive test at 100 km/h conforming to the "Lee-criteria".

#### Introduction

Lee's goal was to find a valid method of estimating the local average power of a signal in the mobile radio environment. His conclusion, that the proper technique is to average 36 samples taken over a distance of 40  $\lambda$  (wavelengths), has become a standard technique, widely used within the industry. This basis have become so widely accepted, that it can sometimes used in situations where Lee sampling is not strictly applicable. Even though it may not be optimum in all situations, it does provide a base-line that allows measurements to be compared. Other experts, such as Professor David Parsons, have suggested other numbers of samples and distances as being a better approximation.

- [1] Estimate of Local Average Power of a Mobile Radio Signal, William C.Y. Lee, IEEE Trans. Veh. Tech. Vol VT-34, No. 1, Feb 1985.
- [2] The Mobile Radio Propagation Channel, David Parsons, John Wiley & Sons 1992, ISBN 0 471 96415 8.

## Background to signal level variations

The envelope of a received mobile radio signal is composed of a slow fading signal with a fast fading signal superimposed on it. In many applications it is necessary to measure the local average power of the slow fading signal by smoothing out (or averaging) the fast fading part.

The severe fading experienced by a moving receiver has two major causes:

- 1. The multi-path phenomenon in the radio environment, the mobile antennas are nearly always lower than surrounding structures such as houses, buildings, etc. The signal transmitted from the base station is usually blokked by these surrounding structures and many reflected waves are generated. Summing all of the multi-path waves at the mobile unit results in fast variations in the received signal which is called multi-path fading. It is also called short-term fading or fast fading referring to the short time period during which signals change.
- 2. The path loss fluctuation (or local mean) - the variation of the average signal power as the mobile moves, is called the path loss fluctuation of the signal. This is due to different propagation paths between the base station and the mobile unit moving over different terrain configurations at different times. Since the propagation path is always changing as the mobile moves, the path loss values and hence local average power of the received signal vary. Because it is affected by the location of the mobile moving in real time and it varies slowly, it is called the local mean of the long-term fading

To set the scene, Lee first expressed the relationship between the short-term and long-term fading. He let r(t) be the signal received at the mobile unit, r<sub>0</sub>(t) be the multi-path fading or the short-term fading signal, and m(t) be the local average power or the local mean of long-term fading signal. The relation among these three parameters he expressed as:

$$r(t) = m(t) * r_0(t)$$

The received signal can also be expressed in a spatial domain as long as the vehicle speed, v is known. This equation can be rewritten as follows:

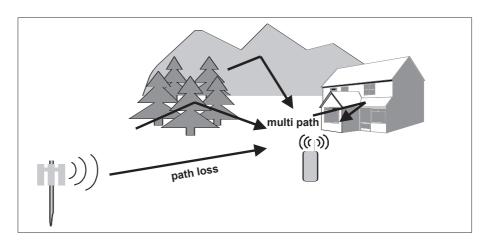
$$r(x) = m(x) \cdot r_0(x)$$

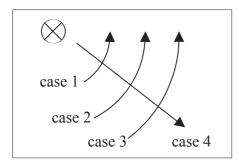
where the distance x is found from the following equation:

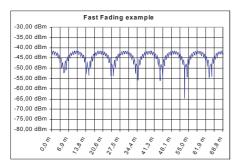
$$x = v * t$$

The local average power or local mean m(x) of a mobile radio signal in real time is a random variable which is affected by the path loss fluctuation. Since the received mobile radio signal contains both short term and long-term fading, to estimate the local mean of long-term fading, we have to try to eliminate the short-term fading  $r_0(x)$ . If we do not process the received signal r(x) correctly, then the true average power and the measured average power will not be the same.

Before calculating the local mean Lee introduced the natural phenomena of the RF signal behaviour in the real world. He considered three cases where the signals are measured along circular roads around the base station at different radii and a fourth case where the signal is received from a direct route to the base station.







- The signal received from case 1 (closest to the transmitter) has a little fading and is mostly under a line-of-sight condition.
- The signals received from cases 2 and 3 start showing severe fading like on the fast fading example picture. The mean power m(x) of this fading signal on a dB scale has a log-normal character. It means that the variable m(x) has an equiprobability of randomness fluctuated around its mean level m(x), which is the mean of the local average signal power.
- In case 4, the local average signal power m(x) is decreasing as the mobile unit moves away from the base station as shown in route figure above. Also the path loss along the distance scale would be proportional to x<sup>-4</sup>.

Lee's goal was to estimate the local mean of a signal in each of the four cases using the same method.

# Obtaining a local average signal power (local mean)

To calculate how to measure the local mean of the signal when the signal is received by a moving receiver Lee addressed two major questions. His approach to both was aimed at reducing the errors in the measurements.

- 1. The first question is how to choose a proper length (2L) of signal data for averaging.
- 2. The second question, after determining the length (2L), is how many independent sample points are needed for averaging over that length.

## Choosing the proper length of a local mean

As we know, the length of a local signal has to be chosen properly. If it is chosen too short, the short-term fading is still present after the averaging process. If it is chosen too long, information about the long-term fading which we want to preserve, will be smoothed out. A correct intermediate length must be chosen.

When averaging over a distance, we are producing an estimate of the true average value. Clearly averages over short distances will show more variation than averages over longer distances where the fast fading will be smoothed out. To find the proper length 2L, Lee calculated the variance of the estimated local mean as a function of the length. It is important to note that he assumed that the fast fading followed Rayleigh statistics. The variance of a set of samples is the square of the standard deviation of the samples from their mean and is a measure of the spread of the sample values. He presented a graph of the variance in dB against 2L.

This graph guides the choice of 2L by showing how much variance we can expect when using different 2Ls. The choice of 2L is a matter of judgement rather than hard fact. Lee suggested the choice of:

- $2L = 20 \lambda$ , if we are willing to accept a  $1 \sigma_m$  spread in a range of 1.56 dB, or
- $2L = 40 \lambda$  if we are willing to accept a  $1 \sigma_m$  spread in a range of 1.0 dB.

If we try to choose less than 20 wavelengths, the 1  $\sigma_m$  spread increases quickly. If we try to choose the length 2L greater than 40  $\lambda$ , the 1  $\sigma_m$  spread decreases very slowly, but, averaging over longer than 40 wavelengths risks smoothing out of long-term fading information. Lee concluded that a length of between 20 wavelengths and 40 wavelengths is the proper length for averaging the signal. It is proper in the sense that a length significantly shorter or longer is likely to result in a reduced accuracy of measurement.

### Sampling average

When using an analogue filter as an averaging process, it is difficult to control the bandwidth and so Lee chose to use arithmetic averaging of samples instead of analogue averaging. This led him to address the question of how many samples should be taken across a length of 2L. Lee aimed to minimise the number of samples and calculated how many points were needed.

The calculation is based upon taking the average of two variables with different statistical distributions. Lee calculates how many samples must be used for the resulting average to be within  $\pm 1$  dB of the true mean. The resulting figure of 36 samples does not guarantee that the average is within  $\pm 1$  dB of the true mean, though it gives a 90% confidence that it will be. The equations could be used to calculate the number of samples needed for greater confidence. Lee chose 36 samples as the figure of  $\pm 1$  dB is similar to the expected 1  $\sigma_m$  spread in a range of 1.0 dB.

In his work Lee says that the samples have to be uncorrelated. He said also that they can be seen uncorrelated if they are at least 0.8  $\lambda$  apart. On choosing 36 samples for 40  $\lambda$  the samples are 1.1  $\lambda$  apart from each other. The sampling and the averaging method have to take care that the sampling distance is always larger than 0.8  $\lambda$ . Otherwise the samples are correlated and this would falsify the average result.

# Effects of different fading environments

Lee concluded that the measured length of a signal necessary to obtain the local average power is in the range of 20 to 40 wavelengths ( $\lambda$ ), based on the Rayleigh distribution. The sufficient number of samples for estimating this local average power values is 36, based on a 90 percent confidence interval and less than 1 dB in error in the estimate. The processed average data retain the long-term fading information which is the local average power of the signal.

Lee's calculations were based on the Rayleigh distribution in which no direct wave component is present. He stated as a worst case you would have line-ofsight like when the mobile is close to the base station. In this case a direct wave component can be received. The direct component produces a different shortterm fading which experiences as Rician distribution. For this distribution we do not need a length as average base of  $2L = 40 \lambda$ . However, when the mobile is moving, the direct wave component at the receiver comes and goes due to the environment. Such a kind of fading can behave different to a fading in a multipath situation. However Lee suggested even in this case it is better to set a length of  $2L = 40 \lambda$ , and N = 36 to handle all situations.

Lee concluded his paper by proposing that this procedure of estimating can be used as a standard method which over time has been demonstrated to be the case.

#### Parsons' results

David Parsons follows a similar process to Lee to calculate the number of samples and length but arrives at different answers. He concludes that with a linear receiver, 85 samples are necessary for a 90% confidence of being within  $\pm 1$  dB. His minimum spacing of samples to ensure that they are independent is 0.38  $\lambda$ , leading to a sampling length 2L of 33  $\lambda$ . He points out that, with a logarithmic receiver, more samples are needed to achieve the same  $\pm 1$  dB to 90%.

#### Conclusion

This paper has tried to give an overview of the basic principles underlying Lee's analysis. In doing so, it has shown that the "Lee-criteria" of 36 samples over 40  $\lambda$ should be treated as a guide line and not as an absolute which cannot be changed. Results should be reviewed in the light of the measurement to be made. Professor Parsons has shown that there are other factors, such as whether the receiver has a linear or logarithmic character, that can affect even a higher numbers of samples needed. Clearly there is more to measuring the local average power of a mobile radio signal than first meets the eye and, no doubt, debate about the best approach will continue.

#### How can Willtek help?

Willtek's Griffin Series Fast Measurement Receivers with extensions cover the range of mobile radio systems from 300 MHz to 2.2 GHz. The cost effective combined receiver and down converter package is rugged and battery powered. The Griffin is able to quickly and accurately perform a wide range of measurement functions in the RF channel.

Up to five downlink channels at speeds of 100 km/h can be measured at 2.2 GHz without jeopardising the "Lee criteria" of valid RF channel measurements. Results can be analysed using Willtek's Hindsite™ software to combine signal levels with topographical information. The results assist in providing high Quality of Service by alignment of the RF prediction model with actual propagation results.

Note: Specifications, terms and conditions are subject to change without prior notice.

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